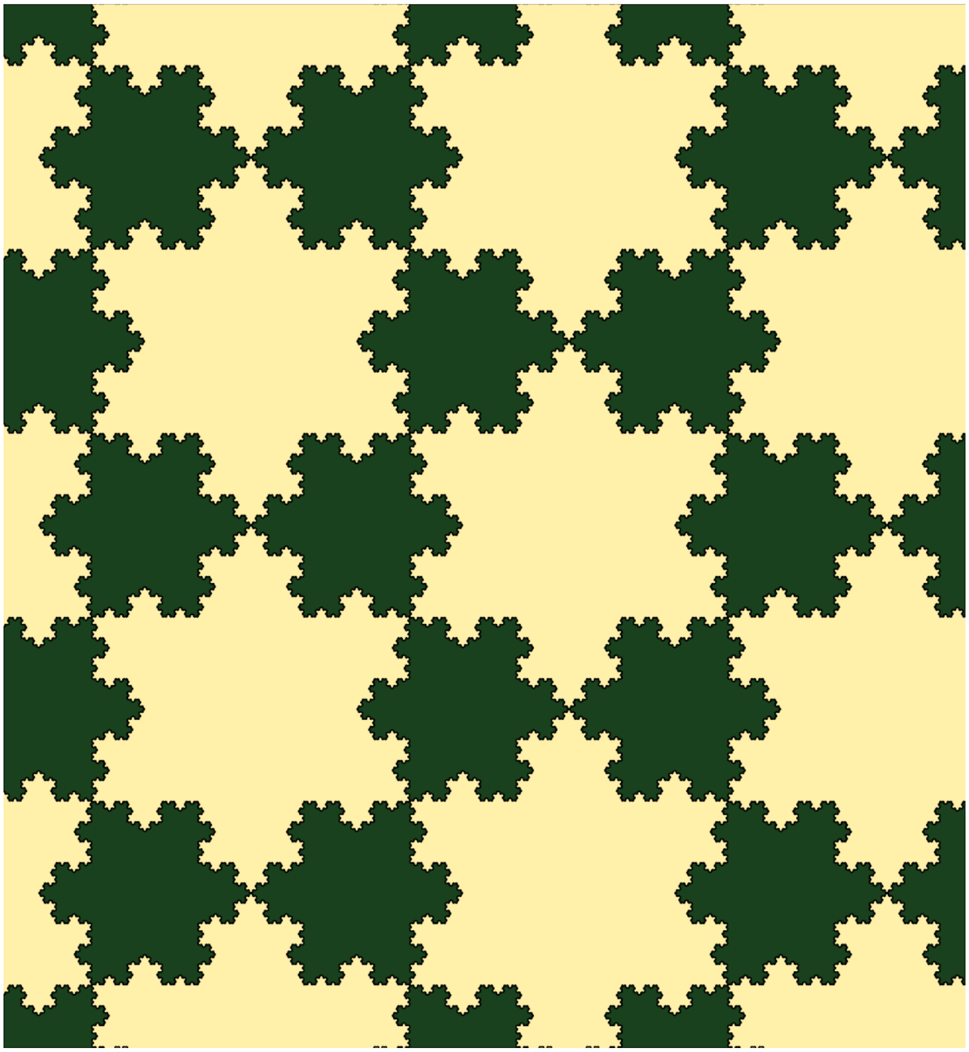
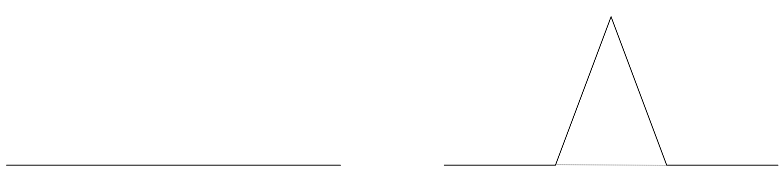


Koch and Levy Curves

The next fractal you will create is the Koch curve, invented by Koch in 1904. The **Koch curve** is the self-similar fractal formed from the Initiator and Generator shown below. Notice that the Generator is formed by dividing the Initiator in thirds and replacing the middle third with two legs of an equilateral triangle whose base is the size of the removed piece. Scale copies of the Koch curve Initiator and Generator, on isometric dot paper, are included on the last page of this lesson.



Koch curve initiator and generator.

1. Create stage 2 of the Koch curve.

2. How many line segments make up Stage 2? If we set our scale so the length of the Initiator is 1 unit, how long is each of the line segments that make up Stage 2? So what is the overall length of Stage 2 of the Koch curve?

3. Create Stage 3 of the Koch curve.

4. How many line segments make up Stage 3? How long is each line segment? So what is the overall length of Stage 3 of the Koch curve?

5. Create stage 4 of the Koch curve.

6. How many line segments make up Stage 4? How long is each line segment? So what is the overall length of Stage 4 of the Koch curve?

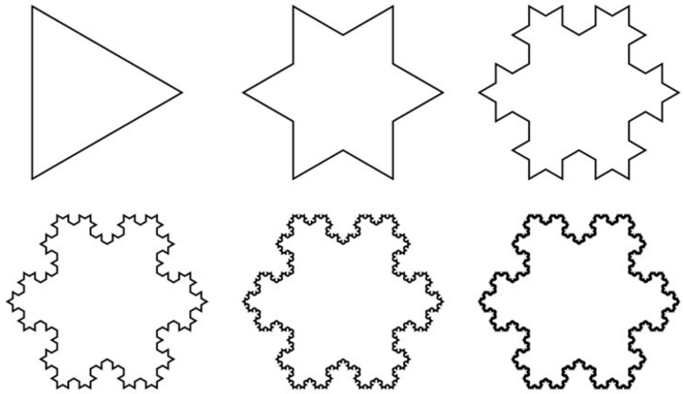
7. Find an algebraic expression for the length of Stage n of the Koch curve.

8. Return to the Initiator and Generator. Explain how interpret your expression for the length of Stage n of the Koch curve in terms of the geometry of the Initiator and Generator.

9. What will the length of the Koch curve be? Explain.

10. Each stage of the Koch curve is simply a collection lines, small one-dimensional pieces. Does it seem like the Koch curve will be one-dimensional? Explain.

On the first page of this lesson, the **Koch snowflake** appeared as an illustration. The stages in its construction are shown below.



Stages in the construction of the Koch snowflake.

11. Explain how the boundary of the Koch curve can be constructed by copies of the Koch curve.

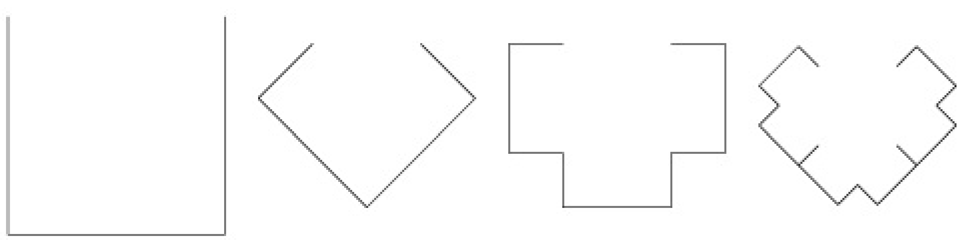
12. What is the minimum number of copies of the Koch curve that can be used to construct the boundary of the Koch snowflake? Explain.,

Assuming that the sides of the equilateral triangle in Stage 0 are *s* = 1, the area of the Koch snowflake is given by the infinite, *geometric series*

$$A = \frac{\sqrt{3}}{4}\left(1 + \frac{3}{9} + \frac{12}{81} + \frac{48}{729} + \dots\right).$$

Explain how each of the terms in parenthesis in expression for the Koch curve area arises. Explain why, despite this series continuing indefinitely, the area of the Koch snowflake must be finite.

The sum of the infinite series, and thus the area of the Koch snowflake, is $A = \frac{\sqrt{3}}{4}\left(\frac{8}{5}\right)$. So, again, we have a situation where an object we have created has a finite area but an infinite perimeter.



The **Lévy C curve** is the self-similar fractal formed from the Initiator and Generator shown above. Notice that the Generator starts with a straight line. An isosceles triangle with angles of 45°, 90° and

45° is built using this line as its hypotenuse. The original line is then replaced by the other two sides of this triangle.

13. Create the next 2-3 stages of the fractal. (An example of stage 12 appears below, as a guide.)